Pearson Edexcel

Mark Scheme (Results)

Summer 2023

Pearson Edexcel International Advanced Level
In Pure Mathematics P3 (WMA13)
Paper 01

Question Number	Scheme	Marks
1. (a)	$\text{g}(3) = -265, \quad \text{g}(4) = 3104$	M1
	States change of sign, continuous and hence root in $[3, 4]$	A1
		(2)
(b)	$x_2 = \sqrt[6]{1000 - 2\times 3} \qquad = 3.1591$	M1 A1
	$(\alpha =) 3.1589$	A1
		(3)
		(5 marks)

Notes

(a)

M1 Attempts the value of g at 3 and 4 with one correct (accept any value for the other as an attempt). Note narrower ranges are possible but must contain the root and lies in [3,4].

A1 Both values correct with reason (Sign change (stated or indicated) and continuous function) and minimal conclusion (root)

(b)

M1 Attempts to substitute $x_1 = 3$ into the formula. Implied by sight of expression, awrt 3.159

A1 awrt 3.1591

A1 $(\alpha =) 3.1589$ cao - must be to 4 d.p. Do not be concerned about the labelling of the root (x or α etc), mark the final answer of (b)(ii). (Note sight of this value implies the M1 even if x_2 is not seen).

Question Number	Scheme	Marks
2 (a) (i)	$\log_6 T = 4 - 2\log_6 x$	B1
(ii)	E.g. $\log_6 T = 4 - 2\log_6 216 \Rightarrow \log_6 T = 4 - 2\times 3 = -2 \Rightarrow T = ...$	M1
	$\Rightarrow T = 6^{-2} = \dfrac{1}{36}$	A1
		(3)
(b)	$\log_6 T = 4 - 2\log_6 x \Rightarrow T = 6^{4-2\log_6 x}$	M1
	$\Rightarrow T = 6^4 \times 6^{\log_6 x^{-2}}$	dM1
	$\Rightarrow T = \dfrac{1296}{x^2}$	A1
		(3)
		(6 marks)

Notes

Mark the question as a whole. Do not be concerned about part labelling.

(a)(i)

B1 Correct linear equation $\log_6 T = 4 - 2\log_6 x$ (oe) The 4 may be written as $\log_6 1296$

(ii)

M1 Substitutes $x = 216$ into an equation linking T and x arising from a linear equation in the logarithms and proceeds to make T the subject. They may have answered (b) first. Do not be concerned about the process for this mark. May be implied by awrt 0.028 following a correct equation.

A1 Correct value $T = \dfrac{1}{36}$. Do not accept 6^{-2}.

(b)

M1 Makes a first step towards achieving an answer. Use of a correct log rule or law **applied** at some stage in their attempt to eliminate logs from the equation.
As a rule of thumb this can be awarded for e.g.

- application of a power rule $-"2"\log_6 x = -\log_6 x^{"2"}$ or $"4" = \log_6 6^{"4"}$ or $4 \to 6^4$ (note that e.g. $\log_6 T = -2\log_6 x + 4 \to x^{-2} + 6^4$ implies this mark)
- an attempt to make T the subject. E.g. $\log_6 T = "4" - "2"\log_6 x \Rightarrow T = 6^{"4"-"2"\log_6 x}$

dM1 Full and complete method in proceeding from an equation of form $\log_6 T = a + b\log_6 x$ $(a, b \neq 0)$ to an equation of form $T = k \times x^{\pm n}$ or equivalent. All log work must be correct but allow slips on coefficients.

A1 Achieves $T = \dfrac{1296}{x^2}$ or equivalent such as $Tx^2 = 1296$ and isw after a correct answer. Allow 6^4 for 1296.

Note: Allow the M marks if a different letter than T is used, e.g. y. But must be correct in terms of T and x for the A mark.

Question Number	Scheme	Marks
3 (i)	$\frac{d}{dx}\ln\left(\sin^2 3x\right)=\frac{1}{\sin^{2} 3x}\times 2\sin 3x\times 3\cos 3x=6\cot 3x$	M1 A1
		(2)
(ii) (a)	$\frac{d}{dx}\left(3x^2-4\right)^6=36x\left(3x^2-4\right)^5$	M1 A1
		(2)
(b)	$\int x\left(3x^2-4\right)^5 dx=\frac{1}{36}\left(3x^2-4\right)^6$	B1ft
	$\int_0^{\sqrt{2}} x\left(3x^2-4\right)^5 dx=\left[\frac{1}{36}\left(3x^2-4\right)^6\right]_0^{\sqrt{2}}=\frac{1}{36}(2)^6-\frac{1}{36}(-4)^6=-112$	M1 A1cso
		(3)
		(7 marks)

Notes

(i)

M1 Attempts to differentiate a ln function. Award for $\frac{d}{dx}\ln\left(\sin^2 3x\right)=\frac{1}{\sin^2 3x}\times...$ where ... could be 1

An alternative could be $\frac{d}{dx}\ln\left(\sin^2 3x\right)=\frac{d}{dx}2\ln\left(\sin 3x\right)=(2\times)\frac{1}{\sin 3x}\times...$ or $\frac{d}{dx}\ln\left(\frac{1-\cos 6x}{2}\right)=\frac{2}{1-\cos 6x}\times...$

A1 $6\cot 3x$ o.e. such as $\frac{6\cos 3x}{\sin 3x}$ or $\frac{6}{\tan 3x}$ or $6(\tan 3x)^{-1}$ but not $6\tan^{-1}3x$ Accept also $\frac{6\sin 6x}{1-\cos 6x}$ or $\frac{3\sin 6x}{\sin^2 3x}$ and isw after a suitably simplified answer.

Constant terms must be gathered and no uncancelled common factors in numerator and denominator.

(ii) (a)

M1 Achieves $\frac{d}{dx}\left(3x^2-4\right)^6=Ax\left(3x^2-4\right)^5$ where A is a constant which may be 1.

A1 $\frac{d}{dx}\left(3x^2-4\right)^6=36x\left(3x^2-4\right)^5$ oe. Need not be simplified. Isw after a correct answer.

(ii) (b)

B1ft $\int x\left(3x^2-4\right)^5 dx=\frac{1}{36}\left(3x^2-4\right)^6$ or $\frac{1}{A}\left(3x^2-4\right)^6$ following through on their (a) provided it is of the form $\frac{d}{dx}\left(3x^2-4\right)^6=Ax\left(3x^2-4\right)^5$ This may arise from attempts via substitution and can be scored from a restart if (ii)(a) was incorrect. Need not be simplified and isw if simplified incorrectly. Condone notation errors such as unneeded integral signs - mark the expression that is their attempt at the integration.

M1 Substitutes in both limits and subtracts (either way round) into an expression of the form $D\left(3x^2-4\right)^6$ where D is a constant but allow slips such as a missing power if the intention is clear. Sight of the subtraction is sufficient. Implied by the correct answer for their integral if substitution not seen. If using integration by substitution they must be substituting the **correct** limits for their variable.

A1cso $(R =)\ -112$ and isw if they make the answer positive after a correct answer seen.
Note: Answer only with no working at all shown scores no marks. Correct integral must be seen.
Note: Attempts at integration by parts are unlikely to succeed, but if done correctly and achieve the correct form of the answer may score the relevant marks.

Note (ii) may be completed by expansion.
(a)

M1 Requires expansion to form $ax^{12}+bx^{10}+cx^{8}+dx^{6}+ex^{4}+fx^{2}+g$ followed by an attempt to integrate each term (power decreased by 1)

A1 Requires correct derivative. $8748x^{11}-58320x^{9}+155520x^{7}-207360x^{5}+138240x^{3}-36864x$

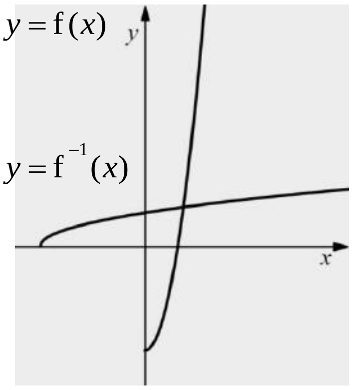
(b)

B1ft Correct answer from a restart, which may be via expansion

$$\frac{81x^{12}}{4}-162x^{10}+540x^{8}-960x^{6}+960x^{4}-512x^{2}$$

M1 Substitutes both limits and subtracts into an expression of the form $ax^{12}+bx^{10}+cx^{8}+dx^{6}+ex^{4}+fx^{2}$

A1cso As main scheme.

Question Number	Scheme	Marks
4.(a)	f $\geqslant -5$	B1 (1)
(b)	 Curve starting on negative *x*-axis and passing through positive *y*-axis, in quadrants 1 and 2 only. Shape and position correct.	M1 A1 (2)
(c)	$2x^2-5=x$ or $2x^2-5=\sqrt{\frac{x+5}{2}}$ or $x=\sqrt{\frac{x+5}{2}}$ or $2\left(2x^2-5\right)^2-5=x$	B1
	Full attempt to solve $2x^2-x-5=0\Rightarrow x=...$ exact	M1
	$x=\frac{1+\sqrt{41}}{4}$	A1
		(3) **6 marks**

Notes

(a) Mark the question as a whole - if (c) answered as (b) allow the marks.

B1 Correct range. Accept $y\geqslant -5,\ f(x)\geqslant -5,\ f\in\left[-5,\infty\right)$ or correct formal set notation but not just $x\geqslant -5$.

(b) Note: if a sketch is redrawn score for the sketch of the inverse only.

M1 For a curve starting on the negative *x*- axis and passing through the positive *y* - axis, in quadrants 1 and 2 only.

A1 Correct shape (curvature) and position. Must be increasing (not bending back on itself) with decreasing gradient, though be tolerant with pen slips at the end. Do not penalise incorrect intercepts.

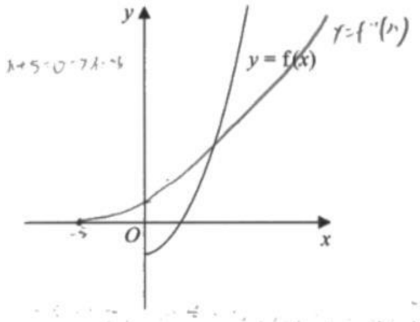
(c)

B1 Sets up a correct equation for the solution, as shown in scheme or equivalents. Should be an equation but allow "=0" implied if there is an attempt to solve. Just $2x^2-x-5$ is B0 with no further working.

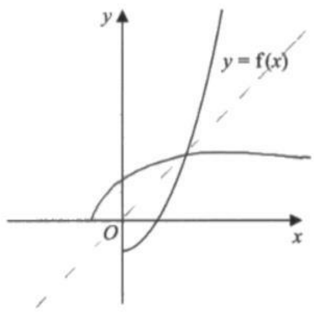
M1 Full attempt to solve a correct equation leading to exact answers.Attempts via $f(x)=f^{-1}(x)$ (oe) will lead a quartic $\left(8x^4-40x^2-x+45=0 \text{ if correct}\right)$ but will likely not lead to exact answers. Note exact answers following a quadratic is fine, but method should be shown for a quartic. Decimal answer only is M0.

A1 $x=\frac{1+\sqrt{41}}{4}$ ONLY.

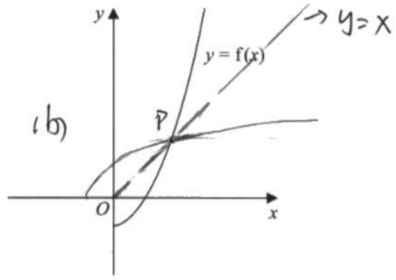
Some examples of curves for question 4(b).



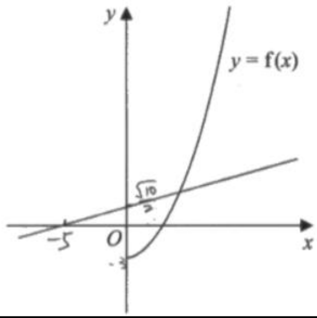
M1A0: Meets conditions for M1 but gradient not decreasing.



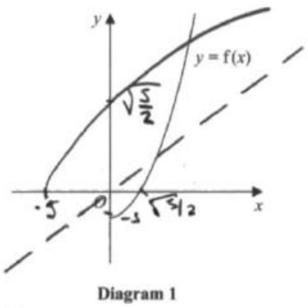
M1A1bod: Curve slips downward at the end due to slip of pen. This is a borderline case.



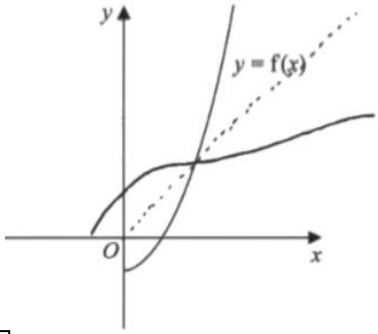
M1A1bod: Gradient is decreasing on the whole.



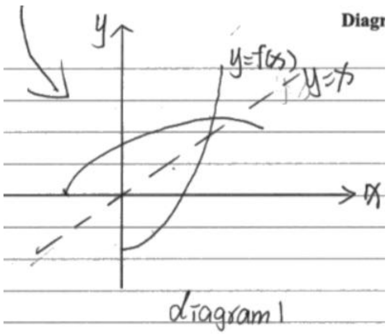
M0A0: Does not start on negative x-axis. Curvature would be unacceptable.



M1A1bod: Position and curvature correct – ignore possible $y = x$ line, mark the curve.



M1A0: Meets condition for M1 but curvature out of tolerance, gradient increases after intersection.



M1A0: Curve is clearly going downward on the right-hand side.

Question Number	Scheme	Marks
5 (i)	States $x = 2$	B1
	$\sqrt{3}\sec x + 2 = 0 \Rightarrow \cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = ...$	M1
	$x = \frac{5\pi}{6}$	A1
		(3)
(ii)	Attempts to use $\cos 2\theta = 1 - 2\sin^2\theta$	M1
	$6\sin^2\theta + 10\sin\theta - 3 = 0$	A1
	$\sin\theta = \frac{-5 \pm \sqrt{43}}{6}\left(= -1.926..., 0.2595...\right) \Rightarrow \theta = \arcsin(...)$	M1
	$\theta = 15.0°, 165°$	A1
		(4)
		(7 marks)

Notes

(i)

B1 States $x = 2$. May be seen anywhere in (i) and don't be concerned where it comes from.

M1 For a correct process to solve $\sqrt{3}\sec x + 2 = 0$ E.g. $\sec x = \frac{1}{\cos x} \Rightarrow \cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = ...$ Allow slips in rearranging but must attempt to solve $\cos x = k, |k| < 1$ or $\sec x = k, |k| > 1$ Degree value ($150°$) following a correct equation implies the M mark. Note some may use $\sec^2 x = 1 + \tan^2 x$ and form a quadratic in tan *x*. These will need a correct identity, correct method to solve a quadratic (which may be by calculator) and attempt to solve $\tan x = k,\ k \neq 0$

A1 $x = \frac{5\pi}{6}$ and no other extra solutions in the range. Accept awrt 2.62 (and isw).

Note that $\sqrt{3}\sec x + 2 = 0 \rightarrow x = \frac{5\pi}{6}$ can score M1A1 as no incorrect work is seen, method implied.

Question required working to be shown $x = \frac{5\pi}{6}$ without seeing at least $\sqrt{3}\sec x + 2 = 0$ extracted first is M0A0.

(ii)

M1 Attempts to use $\cos 2\theta = \pm 1 \pm 2\sin^2\theta$ to form a quadratic equation in $\sin\theta$. If using alternative forms for the identity, must also use $\cos^2\theta = 1 - \sin^2\theta$ before gaining this mark.

A1 Correct 3 term quadratic equation $6\sin^2\theta + 10\sin\theta - 3 = 0$ or a multiple of this. Alternatively may be scored for $6\sin^2\theta + 10\sin\theta = 3$ if followed by completing the square on LHS to solve.

M1 Full attempt to find one value for θ from a quadratic in $\sin\theta$. Must involved

- correct method to solve the quadratic in $\sin\theta$ (usual rules, may use calculator) to produce a value for $\sin\theta$
- use of arcsin(..) to reach the value for θ (you may need to check the values if arcsin(…) is not shown). Radian answers can imply the mark (awrt 0.263, 2.88 if correct).

May be scored from an incorrect identity as long as a quadratic is achieved. Accept arcsin expression for the M

A1 θ =awrt $15.0°, 165°$ and no other solutions in the range. Accept just $15°$ for $15.0°$ (but not awrt $15°$ if it does not round to $15.0°$)

Condone a different variable used than θ throughout.

Question Number	Scheme	Marks
6.(a)	$(2, -10)$	B1 B1
		(2)
(b)	$\text{ff}(0) = \text{f}(-4) = ...$	M1
	$= 8$	A1cso
		(2)
(c)	Attempts to solve $-3(x-2)-10 = 5x+10 \Rightarrow x = ...$	M1
	$x > -\frac{7}{4}$ only	A1
		(2)
(d)	$x(\text{or } \lvert x \rvert) = \frac{16}{3}$	B1
	Attempts $3(\lvert x \rvert - 2) - 10 = 0 \Rightarrow \lvert x \rvert = k,\ k > 0$ or $3(-x-2)-10 = 0 \Rightarrow x = -k$ or $3(x-2)-10 = 0 \Rightarrow x = k \Rightarrow x = -k$	M1
	$x = \left(\frac{16}{3} \text{ and}\right) -\frac{16}{3}$ with no other values	A1
		(3)
		(9 marks)

Notes

(a)

B1 For one correct coordinate

B1 For $(2, -10)$. Allow $x = ..., y = ...$ Do not accept e.g. 6/3 unless 2 has been seen/identified with this.

(b)

M1 For a full attempt at f f (0) . Can be scored for f (-4). Allow for use of their f(0) even if incorrect as long as the process is clear, e.g. f(0) = .. stated or calculated first then used. May be scored by first attempting ff(x) before substituting. This mark is for showing the correct process of composites, so may be scored if there are slips or errors with modulus if the intent is clear.

A1cso $\text{ff}(0) = 8$ only. A0 if other values given.

(c)

M1 Attempts to solve $-3(x-2)-10 = 5x+10 \Rightarrow x = ...$ Allow with equality or any inequality for the M mark.
Alternatively, rearranges to $\lvert x-2 \rvert = ax+b,$ squares both sides and solves the quadratic.

A1 $x > -\frac{7}{4}$ (oe) only. If another inequality or value is given and not rejected withhold this mark.

(d)

Work for (d) must be seen or referred to in (d). Do not accept for work attempted in earlier parts but not used in (d).

B1 For $x = \frac{16}{3}$. Allow when seen even from incorrect working as it could be verified. May be seen on sketch as long as referred to in (d). Allow also for $\lvert x \rvert = \frac{16}{3}$

M1	Correct method to find the root on the negative *x*-axis. E.g. attempts to solve $3\left(\lvert x\rvert - 2\right) - 10 = 0$ to achieve a value for |*x*|, or $3\left(-x-2\right)-10=0$ to achieve a value for *x*, or for reflecting in the *y*-axis (making negative) their $\frac{16}{3}$ from an attempt at $3\left(x-2\right)-10=0$. May be part of longer winded attempts. Allow missing brackets for the M.
Note it is possible to arrive at an equation leading to $x = \pm\frac{16}{3}$ from incorrect starting points, and such methods will score M0.	
A1	For $x = -\frac{16}{3}$ with no other values (aside their $x = \frac{16}{3}$) . Must give the negative value, not just $\lvert x\rvert = \frac{16}{3}$. May be stated on a sketch as long as work seen in (d). Do not isw if they clearly reject this value later or if they try to form an inequality from the values, which is A0 as other values are included.

Question Number	Scheme	Marks
7.(a)	States or implies that $A=2\,500$	B1
	$10\,000=2500\text{e}^{k\times 8}\Rightarrow 8k=\ln 4\Rightarrow k=...$	M1
	$\Rightarrow k=\frac{1}{8}\ln 4$ or awrt 0.1733	A1
		(3)
(b)	$\frac{\text{d}N}{\text{d}t}=60\,000\times -0.6\text{e}^{-0.6\times 5}=-1792$ So decrease is 1790	M1, A1
		(2)
(c)	$60\,000\text{e}^{-0.6t}=2500\text{e}^{0.1733t}$	M1
	$24=\text{e}^{0.1733t+0.6t}\Rightarrow 0.1733t+0.6t=\ln 24\Rightarrow t=...$	dM1
	$T=4.11$	A1
		(3)
		8 marks

Notes

(a)

B1 States or implies that $A=2\,500$. E.g award for $N=2500\text{e}^{kt}$

M1 Attempts to use $N=A\text{e}^{kt}$ with $t=8, N=10\,000$ and their A to set up and solve an equation in k. Correct ln work must be used to solve their equation.
Allow this mark for attempts to find k first by solving simultaneously if they use $t = 1$ for the start of the study: $2500=A\text{e}^{k}, 10000=A\text{e}^{8k}\Rightarrow \text{e}^{7k}=4\Rightarrow 7k=\ln 4\Rightarrow k=...$ but the index and ln work must be correct.

A1 $k=$ awrt 0.1733. Accept the exact value $\frac{1}{8}\ln 4$ and isw after seen.

(b)

M1 $\frac{\text{d}N}{\text{d}t}=C\text{e}^{-0.6\times 5}=...$ where C is a constant. Condone $60\,000\text{e}^{-0.6t}\to 60\,000\text{e}^{-0.6t}$ as long as it is clear they think they have found $\frac{\text{d}N}{\text{d}t}$. Must be correct index (not kt).

A1 Awrt 1790 from a correct derivative. Condone awrt -1790

(c)

M1 Sets $60\,000\text{e}^{-0.6t}=$ their $2500\text{e}^{\text{"}0.1733\text{"}t}$ May use T or another variable instead. Allow a slip on e.g. the 60000 as 6000. Allow with k in place of their "0.1733" as long as they have a value for k in (a).

dM1 Proceeds to rearrange to $e^{mt}=D\ (D>0)$ and applies ln to find t. The ln work must be correct, though there may be slips in the coefficients or index work reaching $e^{mt}=D\ (D>0)$. May be implied by a correct answer for their $e^{mt}=D$
Alternatively, takes ln of both sides first and applies correct ln laws to proceed to make t the subject: $\ln\left(60\,000\text{e}^{-0.6t}\right)=\ln\left(2500\text{e}^{0.1733t}\right)\Rightarrow \ln 60000-0.6t=\ln 2500+0.1733t\Rightarrow t=...$

A1 awrt 4.11 Must be a value, not an expression in ln terms for this mark.

Note Answer only scores no marks, method must be shown and the dM1 must be achieved in order to access the A mark.

Question Number	Scheme	Marks
8. (a)	$f'(x)=6(2x+1)^2 e^{-4x}-4(2x+1)^3 e^{-4x}$	M1 A1
	$=2(2x+1)^2 e^{-4x}\{3-2(2x+1)\}$	dM1
	$=2(2x+1)^2(1-4x)e^{-4x}$	A1
		(4)
(b)	Sets $f'(x)=0\Rightarrow x=-\frac{1}{2},\frac{1}{4}$	B1
	Either $f\left("-\frac{1}{2}"\right)=\ldots$ or $f\left("\frac{1}{4}"\right)=\ldots$	M1
	Both $\left(-\frac{1}{2},0\right)$ and $\left(\frac{1}{4},\frac{27}{8e}\right)$	A1
		(3)
(c)	$\left(\frac{9}{4},\frac{27}{e}\right)$	B1ft B1ft
		(2) **9 marks**

Notes

(a)

M1 Attempts the product rule to achieve $P(2x+1)^2 e^{-4x}\pm Q(2x+1)^3 e^{-4x}$
May also be attempted by the quotient rule - equivalent form after e terms cancel.

A1 $f'(x)=6(2x+1)^2 e^{-4x}-4(2x+1)^3 e^{-4x}$ which may be unsimplified

dM1 Correctly takes out a common factor of $(2x+1)^2 e^{-4x}$ from their expression with an intermediate step before the final answer. Allow if there are minor slips in the $(2x+1)^2 e^{-4x}$ as a factor (such as $(2x+)^2 e^{-4x}$) if recovered - look for the correct remaining terms in the bracket { }. Allow going from an expanded cubic to a factorised form for this mark:
$e^{-4x}\left(2-24x^2-32x^3\right)\rightarrow 2(2x+1)^2(1-4x)e^{-4x}$.

A1 Achieves $2(2x+1)^2(1-4x)e^{-4x}$ with no incorrect algebra. Accept with the brackets in either order.

(b)

B1 $x=-\frac{1}{2},\frac{1}{4}$ o.e. Both required.

M1 Attempts to substitute one of $x=\pm\frac{1}{2},\pm\frac{1}{4}$ into f(x). If substitution not seen may be implied by either of $\left(-\frac{1}{2},0\right)$ or $\left(\frac{1}{4},\frac{27}{8e}\right)$ o.e (accept awrt 1.24 for this mark) or by either of $\left(\frac{1}{2},\frac{8}{e^2}\right)$ (awrt 1.08) or $\left(-\frac{1}{4},\frac{e}{8}\right)$ (awrt 0.340) o.e.

A1 For $\left(-\frac{1}{2},0\right)$ and $\left(\frac{1}{4},\frac{27}{8e}\right)$ o.e. must be exact but isw after exact coordinates given.
Allow as x = .., y =… as long as clearly paired.
Allow the M and A marks if seen in part (c) - mark (b) and (c) together.

(c)

B1ft One correct aspect applied correctly to one of their points. So for either 2 added to one of their x coordinates, or a non-zero y coordinate multiplied by 8. E.g. either $\left(\frac{9}{4},...\right)$ or $\left(...,\frac{27}{e}\right)$ or follow through on $\left("\frac{1}{4}"+2,...\right)$ or $\left(...,8\times"\frac{27}{8e}"\right)$ etc.

B1ft $\left(\frac{9}{4},\frac{27}{e}\right)$ only or follow through on the y coordinate only so $\left(\frac{9}{4},8\times"\frac{27}{8e}"\right)$ (oe) **only**. B0 if another point is given. Accept awrt 9.93 for second ordinate but note 9.92 is a correct follow through on 1.24. Allow as $x = \frac{9}{4}, y = \ldots$.

SC allow B1B0 if coordinates given wrong way round.

Question Number	Scheme	Marks
9(a)	$\frac{\cos 2x}{\sin x}+\frac{\sin 2x}{\cos x}=\frac{\cos 2x}{\sin x}+\frac{2\sin x\cos x}{\cos x}$ (One Correct identity)	B1
	$=\frac{1-2\sin^2 x}{\sin x}+\frac{2\sin x\cancel{\cos x}}{\cancel{\cos x}}$	M1
	$=\frac{1}{\sin x}-\frac{2\sin^{\cancel{2}} x}{\cancel{\sin x}}+2\sin x=\frac{1}{\sin x}=\text{cosec } x$ *	A1*
		(3)
(b)	E.g. Equation is $\text{cosec}^2\theta=6\cot\theta-4\Rightarrow 1+\cot^2\theta=6\cot\theta-4$	M1
	E.g. $\cot^2\theta-6\cot\theta+5=0$	A1
	E.g. $\tan\theta=\frac{1}{5},1$	dM1
	$\theta=0.197,\frac{\pi}{4}$	A1, A1
		(5)
(c)	$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\text{cosec}x\cot x\,dx=\left[-\text{cosec}x\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$	M1
	$=2-\sqrt{2}$	A1
		(2) **10 marks**

Notes

(a) **There are lots of ways of proving this statement. In general score as follows**

B1 For applying at least one CORRECT double or compound angle identity during the proof or for forming a CORRECT single fraction initially.

M1 For a correct overall strategy, e.g. applying double angle identities to reduce terms to single angle arguments and cancelling down terms to eliminate cos *x* terms (score at the stage cos *x* terms could be eliminated), or attempting a single fraction and applying relevant identities to achieve single angle argument with common factor cos *x* in the numerator. Allow slips in signs, such as $\cos 2x=1\pm 2\sin^2 x$ for the M but otherwise identities used must be correct.

A1* Fully correct proof showing all necessary steps, though the left hand side may be implied (and may follow initial lines of aside working). Must see the $\frac{1}{\sin x}\to\text{cosec}\,x$ during the proof . Do not penalise minor notational slips such as missing an *x* in one term.

(b)

M1 Correctly applies the result of (a) and attempts to use relevant identities, allowing sign errors e.g. $\pm 1\pm\cot^2\theta=\text{cosec}^2\theta$ to produce an equation in $\cot\theta$ or other single trig term only. An alternative is

$$\frac{1}{\sin^2\theta}=6\frac{\cos\theta}{\sin\theta}-4\Rightarrow 1=6\sin\theta\cos\theta-4\sin^2\theta\Rightarrow\left(1+4\sin^2\theta\right)^2=36\sin^2\theta\left(1-\sin^2\theta\right)$$

A1 Correct quadratic $\cot^2\theta-6\cot\theta+5=0$ or $5\tan^2\theta-6\tan\theta+1=0$. In the alternative, a correct quadratic in $\sin^2\theta$ or $\cos^2\theta$ e.g. $52\sin^4\theta-28\sin^2\theta+1=0$. The "=0" may be implied by an attempt to solve. May be implied by correct solutions following an unsimplified quadratic.

dM1 Attempts to solve quadratic to find at least one value for their trig term used. Usual rules, may use calculator.

A1 One correct value for θ following from a correct value for the trig term they are working in — must have solved a correct quadratic in the dM. Accept awrt 0.197 or 0.785. Degrees answer are A0A0.

Question Number	Scheme	Marks

A1 Both values correct and no other values in the range. Accept awrt 0.197 and $\frac{\pi}{4}$ only (must be exact but isw after correct value seen).

Note: Allow if a different variable used (such as *x*). For mixed variables allow the M's but only allow the first A (and final A's) if recovered.

Note Answers without working score no marks.

(c)

M1 For using part (a) and achieving $\pm k\,\text{cosec}x$ oe for the integral (limits not required for this mark. May arise from longer methods, but must achieve the correct form.

A1 $2-\sqrt{2}$ Must have scored the M - answer only with no integration shown is M0A0.

Question Number	Scheme		Marks
(a) ALT I	$\frac{\cos 2x}{\sin x}+\frac{\sin 2x}{\cos x}=\frac{\cos 2x\cos x+\sin 2x\sin x}{\sin x\cos x}$	Correct single fraction	B1
	$=\frac{\cos x\left(1-2\sin^2 x\right)+2\sin x\cos x\sin x}{\sin x\cos x}$	Single fraction with single arguments and common factor cos *x* in numerator	M1
	$=\frac{\cancel{\cos x}}{\sin x\cancel{\cos x}}=\frac{1}{\sin x}=\text{cosec }x$ *		A1* **(3)**
(a) ALT II	$\frac{\cos 2x}{\sin x}+\frac{\sin 2x}{\cos x}\equiv\frac{\cos 2x\cos x+\sin 2x\sin x}{\sin x\cos x}$	Correct single fraction	B1
	$\equiv\frac{\cos\left(2x-x\right)}{\sin x\cos x}$	Applies identity to reach single fraction with single arguments and common factor cos *x* in numerator	M1
	$\equiv\frac{\cancel{\cos x}}{\sin x\cancel{\cos x}}\equiv\frac{1}{\sin x}\equiv\text{cosec }x$ *		A1* **(3)**
	Note $\cos(x-2x)$ is equally correct for the M1.		
(a) ALT III	$\frac{\cos 2x}{\sin x}+\frac{\sin 2x}{\cos x}\equiv\frac{\cos^2 x-\sin^2 x}{\sin x}+\frac{2\sin x\cos x}{\cos x}$	Correct identity	B1
	$\equiv\frac{\cos^3 x-\sin^2 x\cos x+2\sin^2 x\cos x}{\sin x\cos x}$	Single fraction with single arguments and common factor cos *x* in numerator	M1
	$\equiv\frac{(\cos^2 x+\sin^2 x)\cancel{\cos x}}{\sin x\cancel{\cos x}}\equiv\frac{1}{\sin x}\equiv\text{cosec }x$ *		A1* **(3)**

Question Number	Scheme	Marks
10 (a)	$x=\dfrac{2y^2+6}{3y-3}\Rightarrow\left(\dfrac{dx}{dy}=\right)\dfrac{4y(3y-3)-3(2y^2+6)}{(3y-3)^2}$	M1 A1
	$\dfrac{dx}{dy}=\dfrac{6y^2-12y-18}{9(y-1)^2}=\dfrac{2y^2-4y-6}{3(y-1)^2}$ o.e	dM1, A1
		(4)
(b)	P and Q are where $\dfrac{dx}{dy}=0$ or where $2y^2-4y-6=0$	B1
	Solves $2y^2-4y-6=0\Rightarrow 2(y-3)(y+1)=0\Rightarrow y=3,-1$	M1
	Subs $y=-1$ and 3 in $x=\dfrac{2y^2+6}{3y-3}\Rightarrow x=..$	dM1
	Achieves $x=-\dfrac{4}{3}$ and $x=4$	A1cso
		(4) **8 marks**

Notes

(a)

M1 Attempts the quotient rule. Condone slips on the coefficients - look for $\dfrac{Ay(3y-3)-B(2y^2+6)}{(3y-3)^2}$

$A,B>0$. Allow a product rule attempt:

$x=(2y^2+6)(3y-3)^{-1}\Rightarrow\left(\dfrac{dx}{dy}=\right)Ay(3y-3)^{-1}+(2y^2+6)\times -B(3y-3)^{-2}$

A1 Correct differentiation which may be unsimplified. Allow if the $\dfrac{dx}{dy}$ is missing or called $\dfrac{dy}{dx}$ for this mark. By product rule $4y(3y-3)^{-1}+(2y^2+6)\times -3(3y-3)^{-2}$ Condone missing brackets if recovered.

dM1 Requires an attempt to get a single fraction with some attempt to simplify.
For the quotient rule look for a simplification of the numerator with like terms collected giving a 3TQ.
Attempts via the product rule will require a correct method to put as a single fraction.

A1 $\left(\dfrac{dx}{dy}=\right)\dfrac{2y^2-4y-6}{3y^2-6y+3}$ or exact simplified equivalent such as $\dfrac{2(y-3)(y+1)}{3(y-1)^2}$ isw after a correct simplified answer. Common factor 3 must have been cancelled. Must be seen in part (a). A0 if called $\dfrac{dy}{dx}$ but allow A1 if LHS is not stated.

Attempts at $\dfrac{dy}{dx}$ can score the first 3 marks if correct. Allow use of x in place of y for the Ms.

(b)

B1 Indicates P and Q are where $\dfrac{dx}{dy}=0$ or where their $2y^2-4y-6=0$ (which may be the denominator of $\dfrac{dy}{dx}$ if they found this instead).

M1 Solves their 3TQ from an attempt at $\dfrac{dx}{dy}=0$ (or denominator of their $\dfrac{dy}{dx}$ =0), usual rules.

dM1 Substitutes both their solutions to $2y^2 - 4y - 6 = 0$ into $x = \frac{2y^2 + 6}{3y - 3}$. Condone slips if the attempt is clear. At least one should be correct if no method is shown.

A1cso Achieves $x = -\frac{4}{3}$ and $x = 4$ only. Must be equations not just values but isw after correct equations seen as long as no contrary work is shown (such as giving horizontal lines). Accept equivalents. Must have come from a correct derivative - though allow from an isw form if a numerical factor was lost in the numerator. Must be exact.

Answers from no working score 0/4 as the question instructs use of part (a), so must see the attempt at setting $\frac{dx}{dy} = 0$

Alt (a)	$x = \frac{2y^2 + 6}{3y - 3} \Rightarrow 3xy - 3x = 2y^2 + 6 \Rightarrow 3x + 3y\frac{dx}{dy} - 3\frac{dx}{dy} = 4y$	M1 A1
	$\frac{dx}{dy} = \frac{4y - 3x}{3(y-1)}$	dM1, A1
		(4)
(b) First 2 marks.	States that P and Q are where $\frac{dx}{dy} = 0$ or where $4y - 3x = 0$	B1
	$\Rightarrow \frac{4}{3}y = \frac{2y^2 + 6}{3y - 3} \Rightarrow 4y^2 - 4y = 2y^2 + 6 \Rightarrow$ as main scheme	M1
Alt II (a)	$x = \frac{2y^2 + 6}{3y - 3} = \frac{2y}{3} + \frac{2}{3} + \frac{8}{3(y-1)} \Rightarrow \frac{dx}{dy} = \frac{2}{3} - \frac{8}{3(y-1)^2}$	M1 A1
	$\frac{dx}{dy} = \frac{2(y-1)^2 - 8}{3(y-1)^2} = \frac{2y^2 - 4y - 6}{3(y-1)^2}$ oe	dM1, A1
		(4)

Notes

(a)

M1 Attempts long division or other method to achieve $Ay + B + \frac{C}{3y - 3}$ oe and differentiates.

A1 Correct differentiation.

dM1 Attempts to get a single fraction and simplifies numerator to 3TQ or uses difference of squares to factorise.

A1 Correct answer.